Discussion on the paper

BOOTSTRAP FOR DEPENDENT DATA: A REVIEW

by

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Congratulations to the authors, this is a very clear and very informative overview on bootstrap for dependent data. Over the past 30 years, research on resampling methods and bootstrap procedures in time series analysis and for dependent data has become a dark thick forest of a variety of approaches. It is very helpful to have this clear and detailed review paper that shed some light on this field of research.

The paper concentrates on univariate stationary processes with short memory, i.e. with absolutely summable autocovariances and it discusses the main important bootstrap methods: block bootstrap, Markovian bootstrap, residual bootstrap and autoregressive sieve bootstrap as the main examples of time domain bootstrap approaches and multiplicative residual bootstrap, local bootstrap and hybrid bootstrap as examples of frequency domain methods.

The paper explains quite well each of these procedures, gives a short outline on their motivation and theoretical background and outlines under which circumstances it can be expected that they work accurately. This is done by using the so-called concept of bootstrap companion processes. If one checks consistency of a bootstrap procedure for a statistic of interest one can compare the asymptotic distribution of the statistic under two settings: under the data generating process of the real world and under the data generating process of the companion process. Consistency of bootstrap requires the identity of these two limiting distributions. This approach provides a simple tool to check validity of bootstrap methods. In the paper this is well illustrated in the discussion of the time domain bootstrap methods and for multiplicative residual bootstrap. We wonder if it could be also used for a more detailed check of the other frequency domain methods. In the paper the pros and cons of these methods are discussed by using more technical arguments.

Inaccuracies of the bootstrap estimate for the distribution of a statistic come from two sources: the type of the data generating process in the bootstrap may differ too strongly from the data generating process in the real world. In particular, this may cause bootstrap inaccuracies if the bootstrap model does not mimic features well that are influential for the distribution of the test statistic. The second source includes inaccurate values for parameters of the bootstrap data generating process. This may be the case for too complex bootstrap procedures. Then the bootstrap model needs too many parameters that must be estimated from the data, explicitly or implicitly. These two components of the bootstrap error correspond to the bias-variance decomposition in nonparametric curve estimation and in model choice. In all these cases, the first statistical aim is to balance these two terms. The first error component, the bias part, is well discussed in the paper. The variance part is not fully addressed in the paper. More theory for the variance part is needed e.g. if one is interested in data adaptive
choice of an optimal or near-optimal bootstrap approach. The question how one chooses the bootstrap procedure with the best bias-variance tradeoff seems to be rather unclear. The choice of bootstrap tuning parameters has been studied, e.g. choice of the block size for block bootstrap. But how to choose data adaptively between several bootstrap procedures seems to be an open research question. This requires the study of new double bootstrap or crossvalidation techniques, perhaps inspired by theoretical work on data-adaptive smoothing parameter selection in nonparametric curve estimation or by recent theoretical progress in model selection.

We also would like to comment further on the bias component of the bootstrap error. Our discussion takes the linear process bootstrap (LPB) as an example. The LPB introduced by McMurry and Politis (2010) uses tapered and banded autocovariance matrix estimators of the whole (univariate) data stretch and i.i.d resampling of appropriately standardized residuals to generate bootstrap observations. Consistency of the LPB was established in McMurry and Politis (2010) for the sample mean. There it was shown that MA processes can be resampled without estimating their coefficients explicitly. This is usually done by numerical optimization or by the innovation algorithm. Both procedures are much more involved than estimating for example AR coefficients. Validity of the LPB, including a multivariate version, is shown in Jentsch and Politis (2011). They show that the multivariate LPB works also for spectral density estimation, but that, in general, it fails for sample autocovariances. With the previous discussion in mind, this is not surprising, because the LPB is designed to capture the covariance structure of the underlying process and the limiting distribution of sample autocovariances is known to depend also on higher order characteristics of the true DGP. Even in the univariate case and under assumed linearity, this is still the case. However, as Jentsch and Politis (2011) show, consistency of the LPB holds for univariate, causal and invertible linear processes for higher order statistics as e.g. autocovariances. At first sight, this may seem to be surprising, because contrary to fully nonparametric bootstrap methods as e.g. the block bootstrap, the LPB mimics by construction only the second order structure of the DGP and not its entire dependence structure. But by taking a closer look and particularly in comparison to the prominent autoregressive sieve bootstrap [cf. the discussion in Kreiss, Paparoditis and Politis (2011)], which is strongly related to the LPB approach, this result fits in the existing literature. Heuristically, the LPB remains valid in this case due to a triangular shape of the matrix $A$ that (approximately) transforms the vector of residuals $(e_1, \ldots, e_n)'$ by pre-multiplication to the vector of observations $(X_1, \ldots, X_n)'$. Thanks to the triangular shape of $A$ and under invertibility, this matrix fits properly together asymptotically with the Cholesky matrix that is involved in the bootstrap algorithm. Eventually, this implies consistency for the LPB in this case. In summary, we see in this example that consistency considerations for bootstrap are often much more involved and complex as one would expect from an analysis using bootstrap companion processes.

To summarize, we think that this review on bootstrap for dependent data gives a very detailed and informative discussion of the state of the art. In our opinion future work is needed for an understanding of the bias-variance performance of bootstrap procedures. Furthermore, this knowledge must be used to develop efficient data adaptive methods that choose between different bootstrap procedures.

References

